**NAME: SOLUTION**

**PROBLEM 1:** Given the state of stress



1. **Determine the stress invariants.**Using MatLab

STRE=[40 40 30; 40 20 0; 30 0 20]

STRE =

40 40 30  
 40 20 0  
 30 0 20

poly(STRE)

ans = 1.0e+004 \*(σ3 -0.0080 σ2 -0.0500 σ3 + 3.4000)

**I1=80MPa  
I2=-500MPa2  
I3=-34,000MPa3**

1. **Determine the principal stresses.**Using MatLab

>> [V P]=eig(STRE)

V =

-0.6340 0.0000 0.7733  
 0.6187 -0.6000 0.5072  
 0.4640 0.8000 0.3804

P =

-20.9902 0 0 **σ1=-21 MPa**  
 0 20.0000 0 **σ2=20 MPa**  
 0 0 80.9902 **σ3=81 MPa**

1. **Determine the direction cosines to each of the principal stresses and calculate θx’x, θx’y, θx’z, θy’x, θy’y, θy’z, θz’x, θz’y, and θz’z.**  
   Using MatLab

>> acos(V)\*180/pi

ans =

129.3450 90.0000 39.3450 **θx’x=129.3θx’y=51.8θx’z=62.4**  
 51.7807 126.8699 59.5231 **θy’x=90.0θy’y=126.9θy’z=36.9**  
 62.3541 36.8699 67.6420 **θz’x=39.3θz’y=59.5θz’z=67.6**

1. **Determine the transformation matrix from the original state of stress to the principal state of stress and prove that it is the transformation matrix by using it to transform the original state of stress.**

>> T=V'

T =

**-0.6340 0.6187 0.4640  
 0.0000 -0.6000 0.8000  
 0.7733 0.5072 0.3804**

>> T\*STRE\*T'

ans =

-20.9902 -0.0000 -0.0000  
 -0.0000 20.0000 0.0000  
 0.0000 0.0000 80.9902

>> P

P =

-20.9902 0 0  
 0 20.0000 0  
 0 0 80.9902

1. **Determine the state of stress defined by rotating x,y plane in the original state of stress through an angle of 30° clockwise about the z axis.**

Using MatLab  
>> T2=[0.866 -.5 0; .5 0.866 0; 0 0 1]

T2 =  
 0.8660 -0.5000 0  
 0.5000 0.8660 0  
 0 0 1.0000

>> T2\*STRE\*T2'

ans =  
 0.3582 28.6582 25.9800 **σx’= 0.4 MPa τx’y’= 28.7 MPa τx’z’= 26.0 MPa**  
 28.6582 59.6391 15.0000 **τx’y’= 28.7 MPa σy’= 60.0 MPa τy’z’= 15.0 MPa**  
 25.9800 15.0000 20.0000 **τx’z’=26.0 MPa τy’z’= 15.0 MPa σz’= 20.0 MPa**

1. **Determine the maximum shear stress for this state of stress.**

Using MatLab

>> (P(1,1)-P(3,3))/2

ans = -50.9902 **τmax=51 MPa**

1. **Determine the transformation matrix that needs to be used to transform the original state of stress to a state of stress that contains the maximum shear stress on two of the faces and a principal state of stress on the third.**

To get to the maximum shear stress state, the original stress state is transformed to the principal state of stress and then the principal state of stress is rotated 45° about the 2-axis. The transformation is [T3]

>> T3=[.7071 0 -.7071; 0 1 0; .7071 0 .7071]

T3 =  
 0.7071 0 -0.7071  
 0 1.0000 0  
 0.7071 0 0.7071

The transformation matrix from the original state of stress to the state of stress where one surface is in the principal state and on the other two surfaces the maximum shear stress exits can not be calculated.

>> T4=T3\*T

**T4 =  
 -0.9951 0.0788 0.0591  
 0.0000 -0.6000 0.8000  
 0.0985 0.7961 0.5971**

This result is proven by first calculating the desired state of stress through a transformation to the principal state of stress and then a second transformation about the 2-axis to the state of maximum shear.

>> T3\*T\*STRE\*T'\*T3'

ans =  
 29.9994 -0.0000 -50.9892  
 -0.0000 20.0000 0.0000  
 -50.9892 0.0000 29.9994

This is compared to the single calculation that was calculated above.

>> T4\*STRE\*T4'

ans =  
 29.9994 -0.0000 -50.9892  
 -0.0000 20.0000 0.0000  
 -50.9892 0.0000 29.9994

The two solutions match; therefore, [T4] is the desired transformation matrix

1. **Draw the Mohr’s circle that defines the bounds for this state of stress.**

**PROBLEM 3:** Determine the transformation matrix for rotating a state of stress on the cube shown to the surface parallel to the following surfaces:

1. CEBG
2. ABEF
3. AEG

Be sure to describe your justification for each of the coordinates used. Draw the transformed coordinate system with respect to the cube.

**The transformation matrix for CEBG**

>> RAbg=[0 0 -2]

RAbg =  
 0 0 -2  
>> RAbc=[-3 1 0]  
RAbc =  
 -3 1 0

>> N=cross(RAbg,RAbc)  
N =  
 2 6 0

>> Uxp=N/norm(N)  
Uxp =  
 0.3162 0.9487 0

>> Uzp=RAbc/norm(RAbc)  
Uzp =  
 -0.9487 0.3162 0

>> Uyp=cross(Uzp,Uxp)  
Uyp =  
 0 0 -1.0000

>> Tp=[Uxp; Uyp; Uzp]  
**Tp =** **0.3162 0.9487 0  
 0 0 -1.0000  
 -0.9487 0.3162 0**

**The transformation matrix for ABEF**

>> RBbf=[0 1 -2]  
RBbf =  
 0 1 -2

>> RBba=[-3 0 0]  
RBba =  
 -3 0 0

>> Npp=cross(RBbf,RBba)  
Npp =  
 0 6 3

>> Uxpp=Npp/norm(Npp)  
Uxpp =  
 0 0.8944 0.4472

>> Uypp=RBbf/norm(RBbf)  
Uypp =  
 0 0.4472 -0.8944

>> Uzpp=RBba/norm(RBba)  
Uzpp =  
 -1 0 0

>> Tpp=[Uxpp; Uypp; Uzpp]  
**Tpp =  
 0 0.8944 0.4472  
 0 0.4472 -0.8944  
 -1.0000 0 0**

**The transformation matrix for AEG**

>> RCag=[3 0 -2]  
RCag =  
 3 0 -2

>> RCae=[0 1 -2]  
RCae =  
 0 1 -2

>> Nppp=cross(RCag,RCae)  
Nppp =  
 2 6 3

>> Uxppp=Nppp/norm(Nppp)  
Uxppp =  
 0.2857 0.8571 0.4286

>> Uyppp=RCag/norm(RCag)  
Uyppp =  
 0.8321 0 -0.5547

>> Uzppp=cross(Uxppp,Uyppp)  
Uzppp =  
 -0.4755 0.5151 -0.7132

>> Tppp=[Uxppp; Uyppp; Uzppp]  
**Tppp =  
 0.2857 0.8571 0.4286  
 0.8321 0 -0.5547  
 -0.4755 0.5151 -0.7132**

**PROBLEM 3:** For the state of stress in Problem 1, determine the state of strain given E=70GPa and ν=0.3.

>> C=[(1/70e9) (-.3/70e9) (-.3/70e9) 0 0 0;  
(-.3/70e9) (1/70e9) (-.3/70e9) 0 0 0;  
(-.3/70e9) (-.3/70e9) (1/70e9) 0 0 0;  
0 0 0 (2\*(1+.3)/70e9) 0 0;  
0 0 0 0 (2\*(1+.3)/70e9) 0;  
0 0 0 0 0 (2\*(1+.3)/70e9)]

C =  
 1.0e-010 \*  
 0.1429 -0.0429 -0.0429 0 0 0  
 -0.0429 0.1429 -0.0429 0 0 0  
 -0.0429 -0.0429 0.1429 0 0 0  
 0 0 0 0.3714 0 0  
 0 0 0 0 0.3714 0  
 0 0 0 0 0 0.3714

>> S=inv(C)  
S =  
 1.0e+010 \*  
 9.4231 4.0385 4.0385 0 0 0  
 4.0385 9.4231 4.0385 0 0 0  
 4.0385 4.0385 9.4231 0 0 0  
 0 0 0 2.6923 0 0  
 0 0 0 0 2.6923 0  
 0 0 0 0 0 2.6923

>> VSTRE=[40e6 20e6 20e6 0 30e6 40e6]'  
VSTRE =  
 40000000  
 20000000  
 20000000  
 0  
 30000000  
 40000000

>> VSTRA=C\*VSTRE  
VSTRA =  
 0.0004000 **εx= 400 με**  
 0.0000286 **εy= 29 με** 0.0000286 **εz=29 με**  
 0 **γzy=0** 0.001143 **γzx=1143 με**  
 0.0014857 **γxy=1486 με**